Control of Angular Motion of a Body by Means of Rotors

M. Z. LITVIN-SEDOY

1 LET US consider a system (G) of rigid bodies, consisting of a free body G_0 and s-kinematic chains of series-connected bodies $G_1^{(\sigma)}, G_2^{(\sigma)}, \ldots, G_{n\sigma}^{(\sigma)}$ $(\sigma = 1,2,\ldots,s)$ (Fig. 1). For all chains each subsequent body $G_\nu^{(\sigma)}$ $(\nu = 2,3,\ldots,n\sigma)$ has, with respect to the previous body $G_{\nu-1}^{(\sigma)}$, up to three angular bodies of freedom, inclusively, and each one of the bodies $G_1^{(\sigma)}$ has, relative to body G_0, up to three angular degrees of freedom. Thus, in the general case the connection between adjacent bodies is realized by means of three-dimensional linkages or gimbals. It is obvious that the system (G) contains the $\sum_{\sigma=1}^{s} n_\sigma + 1$ bodies.

Such a mechanical system is designed for controlling the angular motions of the main body G_0, while the relative angular displacement of each of the bodies $G_k^{(\sigma)}$ $(k = 1,2,\ldots,n\sigma)$ is realized by internal moments of the system (G) (1–3).[1] Furthermore, the relative displacements of certain of the bodies $G_k^{(\sigma)}$ can, generally speaking, be prescribed beforehand. The problem consists of a search for rules governing relative motions of the controlling bodies of the system (G), realizing beforehand the assigned motion of the main body G_0 in an inertial space. We assume here that the principal moment of all external forces with respect to the center of mass C of the entire system (G) will equal zero. The problem is solved with the aid of the integral of moments: $\bar{K} \equiv \bar{K}_0 =$ constant, where $\bar{K}$ is the kinetic moment of system (G) with respect to the center of mass C.

2 We will introduce the right-handed rectilinear coordinate systems: $C_0\xi\eta\zeta$, with the origin at the mass center C_0 of body G_0 and axes which maintain a constant direction in inertial space; $c_0x_0y_0z_0$ are associated with body G_0 (the primary and central axes x_0, y_0, z_0 are in body G_0); $D_k^{(\sigma)}x_k^{(\sigma)}y_k^{(\sigma)}z_k^{(\sigma)}$ are associated with body $G_k^{(\sigma)}$ (axes $x_k^{(\sigma)}, y_k^{(\sigma)}, z_k^{(\sigma)}$ are, in general, neither primary nor central); the origin of $D_\nu^{(\sigma)}$ is selected at a fixed point of body $G_\nu^{(\sigma)}$ with respect to body $G_{\nu-1}^{(\sigma)}$; the origin of $D_1^{(\sigma)}$ in the system of coordinates associated with the body $G_1^{(\sigma)}$ is selected at a point which is fixed with respect to the main body G_0. So long as the relative motion of body $G_k^{(\sigma)}$ appears to be revolving about an axis fixed in the previous body, the origin of $D_k^{(\sigma)}$ is selected at some point lying on this axis. The symbol $C_k^{(\sigma)}$ in Fig. 1 denotes the mass center of body $G_k^{(\sigma)}$; the symbol $M_{kj}^{(\sigma)}$ is the jth mass point of this body. The remaining notations are obvious (i.e., $M_{kn}^{(\sigma)}$ is the nth mass point, etc.).

Let us introduce the following notations: m_0—as the mass of body G_0; $m_k^{(\sigma)}$—the mass of body $G_k^{(\sigma)}$:

$$l_k^{(\sigma)} = \begin{Vmatrix} -A_k^{(\sigma)} & I_{xyk}^{(\sigma)} & I_{xzk}^{(\sigma)} \\ I_{yxk}^{(\sigma)} & -B_k^{(\sigma)} & I_{yzk}^{(\sigma)} \\ I_{zxk}^{(\sigma)} & I_{zyk}^{(\sigma)} & -C_k^{(\sigma)} \end{Vmatrix}$$

as the matrix of moments of inertia for body $G_k^{(\sigma)}$ with respect to axes $D_k^{(\sigma)}x_k^{(\sigma)}, D_k^{(\sigma)}y_k^{(\sigma)}, D_k^{(\sigma)}z_k^{(\sigma)}$ associated with it; A_0, B_0, C_0 are the moments of inertia for the main body G_0 with respect to axes C_0x_0, C_0y_0 and C_0z_0, respectively; $f_{k-1}^{(\sigma)}$ is the row matrix for projections of vector $\bar{a}_{k-1}^{(\sigma)} = \overline{D_{k-1}^{(\sigma)}D_k^{(\sigma)}}$ $(k = 1,2,\ldots,n\sigma)$ on axes $x_{k-1}^{(\sigma)}, y_{k-1}^{(\sigma)}, z_{k-1}^{(\sigma)}$; $C_k^{(\sigma)}$ is the row matrix for the projections of the radius vector $\bar{c}_k^{(\sigma)} = \overline{D_k^{(\sigma)}C_k^{(\sigma)}}$ on the axes $x_k^{(\sigma)}, y_k^{(\sigma)}, z_k^{(\sigma)}$; $\theta_{kj}^{(\sigma)} = \|x_{kj}^{(\sigma)}, y_{kj}^{(\sigma)}, z_{kj}^{(\sigma)}\|$ s the row matrix of the coordinate points $M_{kj}^{(\sigma)}$ of the body $G_k^{(\sigma)}$; $m_{kj}^{(\sigma)}$ is the mass of the material point $M_{kj}^{(\sigma)}$:

$$l_{\lambda\mu}^{(\sigma)} = \|l_{gh}^{(\lambda\mu,\sigma)}\| \qquad (g,h = 1,2,3;\ \lambda,\mu = 0,1,2,\ldots,n_\sigma)$$

the matrix of the direction cosines of trihedron $x\lambda^{(\sigma)}y\lambda^{(\sigma)}z\lambda^{(\sigma)}$ with respect to $x\mu^{(\sigma)}, y\mu^{(\sigma)}, z\mu^{(\sigma)}$:

$$[l_{12}^{(\lambda\mu,\sigma)} = \cos(x_\lambda^{(\sigma)}, y_\mu^{(\sigma)}) \text{ etc.}]$$

where the values $\lambda = 0$ or $\mu = 0$ refer to trihedron $x_0y_0z_0$; $\omega_{\lambda\mu}^{(\sigma)} = \|p_{\lambda\mu}^{(\sigma)}, q_{\lambda\mu}^{(\sigma)}, r_{\lambda\mu}^{(\sigma)}\|$ is the matrix for projections of the instantaneous angular velocity of body $G_\lambda^{(\sigma)}$ with respect to body $G_\mu^{(\sigma)}$ on the axes, associated with the same body $G_\lambda^{(\sigma)}$. By p, q, and r we designate the projections of the absolute instantaneous angular velocity of body G_0 on the respective axes x_0, y_0, and z_0 associated with it, respectively.

We specify the angular position of body G_0 in the inertial space with the aid of Euler angles ψ, ϑ, γ (Fig. 2). We define the angular position of body $G_\lambda^{(\sigma)}$ with respect to the body $G_\mu^{(\sigma)}$ similarly with the aid of Euler angles $\psi_{\lambda\mu}^{(\sigma)}$, $\vartheta_{\lambda\mu}^{(\sigma)}$, $\gamma_{\mu\lambda}^{(\sigma)}$ (By $G_0^{(\sigma)}$ it is understood to mean body G_0.)

The angular velocities are associated with the Euler angles by the Euler kinematic relationship

$$\begin{aligned} p_{\lambda\mu}^{(\sigma)} &= \dot{\gamma}_{\lambda\mu}^{(\sigma)} + \dot{\psi}_{\lambda\mu}^{(\sigma)} \sin\vartheta_{\lambda\mu}^{(\sigma)} \\ q_{\lambda\mu}^{(\sigma)} &= \dot{\vartheta}_{\lambda\mu}^{(\sigma)} \sin\gamma + \dot{\psi}_{\lambda\mu}^{(\sigma)} \cos\vartheta_{\lambda\mu}^{(\sigma)} \cos\gamma_{\lambda\mu}^{(\sigma)} \\ r_{\lambda\mu}^{(\sigma)} &= \dot{\vartheta}_{\lambda\mu}^{(\sigma)} \cos\gamma_{\lambda\mu}^{(\sigma)} - \dot{\psi}_{\lambda\mu}^{(\sigma)} \cos\vartheta_{\lambda\mu}^{(\sigma)} \sin\gamma_{\lambda\mu}^{(\sigma)} \end{aligned} \qquad [1]$$

Finally, with the aid of symbol $\sim$ let us agree to denote an operator transforming a row matrix $a = \|a_1, a_2, a_3\|$ to a special matrix (4):

$$\tilde{a} = \begin{Vmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{Vmatrix}$$

The relationships are valid:

$$\dot{l}_{\lambda\mu}^{(\sigma)} = -\tilde{\omega}_{\lambda\mu}^{(\sigma)} l_{\lambda\mu}^{(\sigma)} \qquad (\lambda,\mu = 0,1,2,\ldots,n_\sigma) \quad (\sigma = 1,2,\ldots,s) \qquad [2]$$

$$l_{k0}^{(\sigma)} = \prod_{x=0}^{k-1} l_{(k-x)(k-x-1)}^{(\sigma)} \qquad (k = 1,2,\ldots,n_\sigma) \qquad [3]$$

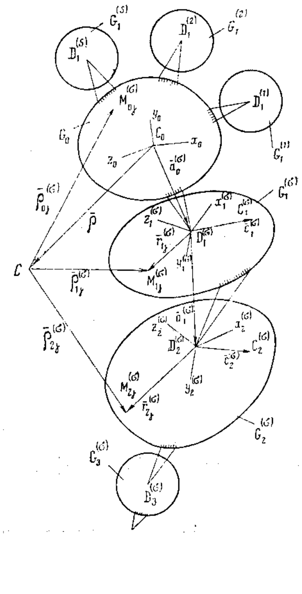

Fig. 1

Translated from Vestnik Moskovskogo Universiteta (Bulletin of Moscow University), no. 6, 72–79 (1960). Presented by the author at the first All-Union Congress on Theoretical and Applied Mechanics, February 2, 1960. Translated by Primary Sources, New York. Extensive revision of the translation kindly supplied by E. R. Dunkel of International Business Machines Corporation, Owego, N. Y.

[1] Numbers in parentheses indicate References at end of paper.

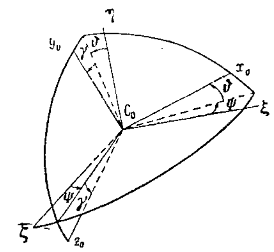

Fig. 2

K is used to denote the row matrix of the projections on axes, x_0, y_0, z_0 associated with the main body of the kinetic moment $\bar{K}$ of system (G) with respect to its mass center C; $K_k^{(\sigma)}$ is the row matrix for projections on the same axes x_0, y_0, z_0 of the kinetic moment of body $G_k^{(\sigma)}$ with respect to the mass center C of system (G); $K^{(0)}$ is the corresponding row matrix for the main body.

3 In order to find kinetic moment K we determine the radius vector $\bar{\rho}$ of mass center C with respect to the mass center C_0 of the main body. Expressing the sum of static moments in matrix form and using Eq. [3], we find row matrix ρ of the projections of vector $\bar{\rho}$ on axes of the system x_0, y_0, z_0:

$$\rho = \frac{1}{m} \sum_{\sigma=1}^{s} \sum_{k=1}^{n\sigma} (M_k^{(\sigma)} f_{k-1}^{(\sigma)} + m_k^{(\sigma)} c_k^{(\sigma)} l_{k-1}^{(\sigma)}) \times \prod_{x=1}^{k-1} l_{(k-x)(k-x-1)} \quad [4]$$

where

$$m = m_0 + \sum_{\sigma=1}^{s} \sum_{k=1}^{n\sigma} m_k^{(\sigma)}$$

is the mass of system (G)

$$M_k^{(\sigma)} = \sum_{\lambda=k}^{n\sigma} m_\lambda^{(\sigma)}$$

4 The kinetic moment of system (G) relative to its center of mass is determined by expression

$$K = \|K_x K_x K_z\| = K^{(0)} + \sum_{\sigma=1}^{s} \sum_{k=1}^{n\sigma} K_k^{(\sigma)} \quad [5]$$

Starting from the definition of the kinetic moment and taking Eq. [2] into account, we write that

$$K_k^{(\sigma)} = -\sum_{G_k^{(\sigma)}} m_{kj}^{(\sigma)} V_{kj}^{(\sigma)} \widetilde{\chi_{kj}^{(\sigma)}}$$

where $\chi_{kj}^{(\sigma)}$ is the row matrix of projections on axes x_0, y_0, z_0 of radius-vector $\rho_{kj}^{(\sigma)} = CM_{kj}^{(\sigma)}$ of the jth mass point for body $G_k^{(\sigma)}$ with respect to the mass center C of system (G), $V_{kj}^{(\sigma)}$ is the row matrix of projections on these same axes with derivative $d\bar{\rho}_{kj}^{(\rho)}/dt$.

It is obvious from Ref. 4 that

$$V_{kj}^{(\sigma)} = \dot{\chi}_{kj}^{(\sigma)} - \chi_{kj}^{(\sigma)} \tilde{\omega} = \dot{\chi}_{kj}^{(\sigma)} + \omega \widetilde{\chi_{kj}^{(\sigma)}}$$

that is

$$K_k^{(\sigma)} = -\sum_{G_k^{(\sigma)}} m_{kj}^{(\sigma)} [\dot{\chi}_{kj}^{(\sigma)} \widetilde{\chi_{kj}^{(\sigma)}} + \omega(\widetilde{\chi_{kj}^{(\sigma)}})^2] \quad [6]$$

As follows from Fig. 1

$$\bar{\rho} + \bar{\rho}_{kj}^{(\sigma)} = \sum_{x=1}^{k} \bar{a}_{x-1}^{(\sigma)} + \bar{r}_{kj}^{(\sigma)}$$

where $\overline{r_{kj}^{(\sigma)}} = \overline{D_k^{(\sigma)} M_{kj}^{(\sigma)}}$ is the radius-vector of point $M_{kj}^{(\sigma)}$ with respect to the origin of $D_k^{(\sigma)}$. Using $R_{k-1}^{(\sigma)}$ to denote the row matrix

$$R_{k-1}^{(\sigma)} = \sum_{x=1}^{k} f_{x-1}^{(\sigma)} l_{(x-1)\sigma} - \rho \quad [7]$$

(where $\chi = 1$, $l_{00} = E$ is a unitary matrix) we write matrix $\chi_{kj}^{(\sigma)}$ in the form

$$\chi_{kj}^{(\sigma)} = R_{k-1}^{(\sigma)} + \theta_{kj}^{(\sigma)} l_{k0}^{(\sigma)} \quad [8]$$

By replacing the quantity $\chi_{kj}^{(\sigma)}$ in Eq. [6] with its expression [8], we obtain, after transformation

$$K_k^{(\sigma)} = -m_k^{(\sigma)} \{ (\dot{R}_{k-1}^{(\sigma)} + c_k^{(\sigma)} l_{k0}) \widetilde{R_{k-1}^{(\sigma)}} + \dot{R}_{k-1}^{(\sigma)} (\widetilde{c_k^{(\sigma)} l_{k0}^{(\sigma)}}) + \omega [(\widetilde{R_{k-1}^{(\sigma)}})^2 + (\widetilde{c_k^{(\sigma)} l_{k0}^{(\sigma)}}) \widetilde{R_{k-1}^{(\sigma)}} + \widetilde{R_{k-1}^{(\sigma)}} (\widetilde{c_k^{(\sigma)} l_{k0}^{(\sigma)}})] \} - (\omega_{k0}^{(\sigma)} + \omega l_{k0}^{(\sigma)T}) I_k^{(\sigma)} l_{k0}^{(\sigma)} \quad [9]$$

where m is the symbol of the transposed matrix. The kinetic moment $\bar{K}$ is determined by the summation [5] in which the matrix ρ [4] enters through matrix $R_{k-1}^{(\sigma)}$ [7].

It is always possible to diagonalize matrices $I_k^{(\sigma)}$ ($k = 1,2, \ldots, n\sigma$) of the moments of inertia, adjusting the associated axes, $x_k^{(\sigma)}$, $y_k^{(\sigma)}$, $z_k^{(\sigma)}$ along the principal axes to point $D_k^{(\sigma)}$. However, if it is necessary to determine the influence of deviations in the real mass parameters of bodies $G_k^{(\sigma)}$ of the system on those calculated on the basis of the dynamics of the system (G), it is advisable to maintain the reading directions, corresponding to the principal axes of the undistorted structure.

5 Let us specify the requirements of angular motion for body G_0:

$$\psi \equiv \psi(t) \qquad \vartheta \equiv \vartheta(t) \qquad \gamma \equiv \gamma(t) \quad [10]$$

The projections K_x, K_y, K_z in [5] of kinematic moment $\bar{K}$ on the axes associated with the main body are expressed with the aid of the first three integrals in Ref. 5:

$$K_x = K_0 \sin\vartheta \qquad K_y = K_0 \cos\gamma \cos\vartheta$$
$$K_z = -K_0 \sin\gamma \cos\vartheta \quad [11]$$

Furthermore

$$p = \dot{\psi} \sin\vartheta + \dot{\gamma} \qquad q = \vartheta \sin\gamma + \dot{\psi} \cos\gamma \cos\vartheta$$
$$r = \dot{\vartheta} \cos\dot{\gamma} - \dot{\psi} \cos\vartheta \sin\gamma \quad [12]$$

As soon as the functions in [10] are given, the first integrals in [11] determine, with computation of Eqs. [1] and [12], three relations (in general, differential) between laws of relative angular motions for bodies $G_k^{(\sigma)}$; where there is suitable selection of controlling bodies, they provide for the rotation of body G_0 according to relations [10].

Matrix $\omega_{k0}^{(\sigma)}$, which characterizes the angular velocity of body $G_k^{(\sigma)}$ relative to body G_0, is expressed in terms of the matrix of intermediate angular velocities with the aid of relationship

$$\varpi_{k0}^{(\sigma)} = \varpi_{kp}^{(\sigma)} + l_{kp}^{(\sigma)} \varpi_{p0}^{(\sigma)} l_{kp}^{(\sigma)T}$$
$$(k,p = 1,2, \ldots, n_\sigma) \quad [13]$$

In fact, by virtue of Eq. [3]

$$\dot{l}_{k0}^{(\sigma)} = \dot{l}_{kp}^{(\sigma)} l_{p0}^{(\sigma)} + l_{kp}^{(\sigma)} \dot{l}_{p0}^{(\sigma)}$$

and it follows from Eq. [2] that

$$-\varpi^{(\sigma)} l_{k0}^{(\sigma)} = -\varpi_{kp}^{(\sigma)} l_{kp}^{(\sigma)} l_{p0}^{(\sigma)} - l_{kp}^{(\sigma)} \varpi_{p0}^{(\sigma)} l_{k0}^{(\sigma)T}$$

which also leads to relationship [13].

As regards matrices of the type $\widetilde{c_k^{(\sigma)} l_{k0}^{(\sigma)}}$, then $\widetilde{c_k^{(\sigma)} l_{k0}^{(\sigma)}} = l_{k0}^{(\sigma)T} \widetilde{C_k^{(\sigma)}} l_{k0}^{(\sigma)}$.

6 As an example, let us consider system (G) of five bodies G_0, $G_1^{(\sigma)}$ $(\sigma = 1, 2, 3, 4 = s)$. Controlling bodies $G_1^{(\sigma)}$, $G_1^{(2)}$, $G_1^{(3)}$ rotate about axes x_0, y_0, z_0 of the main body G_0, respectively; the center of the mass $C_1^{(g)}$ of body $G_1^{(g)}$ $(g = 1, 2, 3)$ lies on the axis of its relative rotation. Then

$$\psi_{10}^{(g)} = \vartheta_{10}^{(g)} = 0 \qquad \dot{\gamma}_{10}^{(g)} = \Omega_g$$

$$f_0^{(1)} = \|s_1^{(1)}, 0, 0\|$$

$$f_0^{(2)} = \|0, s_1^{(2)}, 0\|$$

$$f_0^{(3)} = \|0, 0, s_1^{(3)}\|$$

where

$$s_1^{(g)} = (C_0C_1^{(g)})$$

Body $G_1^{(4)}$ has two degrees of freedom relative to body G_0

$$\psi_{10}^{(4)} = \alpha \qquad \vartheta_{10}^{(4)} = \beta \qquad \gamma_{10}^{(4)} \equiv 0$$

The law for $\alpha(t)$ and $\beta(t)$ of the relative motion of body $G^{(4)}$ was given a priori.

The mass center $C_1^{(4)}$ of body $G_1^{(4)}$ coincides with the fixed point $D_1^{(4)}$ of this body in the main body G_0, lying on axis y_0 at a distance of $s \neq s_1^{(2)}$ from the origin of C_0. The mass center C of the whole system (G) coincides with the mass center C_0 of the body G_0. Let us accept $x_1^{(\sigma)}$, $y_1^{(\sigma)}$, $z_1^{(\sigma)}$ as principal axes. Bodies $G_1^{(\sigma)}(\sigma = 1, 2, 3, 4)$ are essentially bodies of rotation with respect to axes $x_1^{(\sigma)}$, respectively, $B_1^{(\sigma)} = C_1^{(\sigma)} = \ni_1^{(\sigma)}$; $\ni_1^{(\sigma)}$ is the equatorial central moment of inertia for body $G_1^{(\sigma)}$, $c_0 = c_1^{(g)} = 0$.

Consequently

$$f_0^{(4)} = \|0, s, 0\| \qquad s = (C_0D_1^{(4)}) \qquad \rho = 0 \qquad R_0 = f_0$$

It is evident that

$$K^{(0)} = -\omega I_0 = \|A_0p, B_0q, C_0r\|$$

Furthermore, using relationship [9], we obtain

$$K_1^{(g)} = -m_1^{(g)}\omega\widetilde{(f_0^{(g)})}^2 - (\omega_{10}^{(g)} + \omega l_{10}^{(g)T})I_1^{(g)}\ l_{10}^{g}$$

$$(g = 1, 2, 3)$$

$$K_1^{(4)} = -[m_1^{(4)}\omega\widetilde{(f_0^{(4)})}^2 + (\omega_{10}^{(4)} + \omega l_{10}^{(4)T})I_1^{(4)}l_{10}^{(4)}]$$

Here (Fig. 2)

$$l_{10} = \|l_{gh}^{(10,4)}\| = \left\|\begin{matrix} \cos\alpha\cos\beta & \sin\beta & -\sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & \sin\alpha\sin\beta \\ \sin\alpha & 0 & \cos\alpha \end{matrix}\right\|$$

and by virtue of the relationships in [1], considered when $\lambda = 1$, $\mu = 0$, $\sigma = 4$:

$$p_{10}^{(4)} = \dot{\alpha}\sin\beta \qquad q_{10}^{(4)} = \dot{\alpha}\cos\beta \qquad r_{10}^{(4)} = \beta$$

$$(\omega_{10}^{(4)} = \|p_{10}^{(4)}, q_{10}^{(4)}, r_{10}^{(4)}\|)$$

Using Eqs. [5] and [9], we find the relationship

$$\Omega_1^{(1)}(t) = \frac{1}{A_1^{(1)}}\{K_0\sin\vartheta - [(A_1^{(4)} - \ni_1^{(4)})\cos^2\alpha\cos^2\beta + \ni_1^{(2)} + \ni_1^{(3)} + \ni_1^{(4)} + A_0 + A_1^{(1)} + m_1^{(2)}(s_1^{(2)})^2 + m_1^{(3)}(s_1^{(3)})^2 + m_1^{(4)}s^2]p - (A_1^{(4)} - \ni_1^{(4)})(q\sin\beta - r\sin\alpha\cos\beta)\cos\alpha\cos\beta - \ni_1^{(4)}\beta\sin\alpha = (\tfrac{1}{2})(A_1^{(4)} - \ni_1^{(4)}\dot{\alpha}\cos\alpha\sin\beta\}$$

governing the laws of natural rotations (and two analogies)

$$\Omega_1^{(1)}(t),\ \Omega_1^{(2)}(t),\ \Omega_1^{(3)}(t)$$

for controlling bodies $G_1^{(g)}$, providing for a fixed rotation of the main body G_0 when there is a fixed relative motion of body $G_1^{(4)}$.

—*Submitted February 2, 1960*

References

1 Litvin-Sedoy, M. Z., "On a complex gyroscopic effect," Vestn. Mosk. Univ. (Bulletin of the Moscow University), no. 10 (1954).

2 Litvin-Sedoy, M. Z., "Equations for the motion of bodies, individual parts of which have one degree of relative mobility," Uch. Zap. Mosk. Gos. Univ., Mekhanika (Scientific Notes of the Moscow State University, Mechanics) (1954).

3 Roberson, R. E., "A review of the current status of satellite altitude control," J. Astronaut. Sci. **6**, 25–39 (1959).

4 Frazer, R., Duncan, V. and Collar, A., *Theories of Matrices and Their Application* (Foreign Literature Publishing House, 1950).

5 Golubev, V. V., *Discourse on Integrating Equations for the Motion of Rigid Bodies Around a Fixed Point* (Gostekhizdat, State Publishing House of Theoretical and Technical Literature, 1953).

Reviewer's Comment

The description as developed by this article deals with the angular control of a so-called "free" rigid body by the employment of the dynamic reaction of rigid kinetic appendages, sometimes known as "inertia wheels." Exception is taken here to the term "free body" which is used to describe G_0. More appropriately it should be "controlled body." The author goes on to state that there are s-kinematic chains of series-connected bodies attached to the "free body." In almost the same breath he speaks of each of the bodies as having mass points, total mass, and inertia which is not consistent with the definition of kinematics.

Careful study of Fig. 1 reveals that all bodies with subscript n are attached to a body with subscript $n - 1$. The superscript σ denotes the number of bodies so attached. It is entirely possible that there may be members of different chains which would have the same sub- and superscript notation. This could be rather confusing. This configuration in Fig. 1 reminds one of a crystallographic array of molecules in a solid state.

It may be noted here that *directional inertial reference* apparently implies a Galilean frame (nonrotating axes and an unaccelerated origin). It is not apparent that the characterizations of this paper are completely applicable to the directionally inertial relative coordinates common in celestial mechanics and astrodynamics (axes directionally fixed, but the origin of the triad may experience acceleration). It would not be too difficult to relate this work to past American literature on the structural feedback problem as related to rocket structures with gimbal-mounted rocket motors or hydrogen-peroxide control jets.

—E. R. Dunkle
International Business Machines Corp.